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Stability of boiling systems

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Abstract—A rigorous approach for the stability analysis of boiling systems under the assumption of uniform boiling on the heated surface is introduced. The semi-analytical procedure is based on linear stability analysis of general distributed parameter systems. For experiments controlled by means of a stabilizing fluid or by feedback temperature control, parameter domains ensuring stable operation can be computed and systematically maximized in view of all relevant system components including heater, power supply, controller, sensors and filters. Stability limitations arising from power supply saturation are quantified by numerical stability analysis and dynamic simulation. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

For transition boiling, the negative slope of the boiling curve prevents steady-state measurements for heat flux controlled experiments. Stable operation can only be achieved by temperature control of the wall superheat in one of the following ways: passive stabilization by means of fluid heating, active stabilization by adjusting the energy input of an electrical heat source according to some control law, or a combination of the two. The success of the employed method depends on a number of circumstances including geometry and material of the apparatus, the characteristics of the heat transfer to the boiling liquid and from the stabilizing fluid, sensor location and dynamics, as well as the chosen control algorithm. The aim of this study is to outline a rigorous stability analysis of boiling systems and suggest an approach to a systematic design of stable steady-state experiments in view of all of the above-mentioned factors.

Only systems with uniform temperature distribution across the boiling surface (i.e. only one boiling regime, as for instance nucleate boiling, exists on this surface at a given time) are to be considered here. Uniform boiling always is a simplifying assumption, since it neither accounts for the strong local temperature and heat flux fluctuations resulting from oscillating wetting conditions (especially in the transition boiling regime [1]), nor does it consider the possible formation of 'dry patches' observed during high heat flux nucleate boiling [2, 3]. The stability and propagation of heterogeneous temperature profiles, well known as 'two-mode' boiling from temperature-controlled experiments with wire or thin heaters in both pool or forced convective boiling, is not the subject of this investigation.

Stability analysis of boiling experiments has mostly been studied in conjunction with a particular method of stabilization for the experimental apparatus at hand. Only few studies deal with stability issues in a more general context. The first attempts of stabilized experiments can be summarized as follows: Poletarkin *et al.* [4] first proposed the use of stabilizing fluids for temperature control in boiling experiments. Independently, with an experimental design typical for stabilizing fluids, Ellion [5] used water at high pressure flowing through a tube heated by electric current to record boiling curves of water at moderate pressures. Later experiments employed other stabilizing fluids like liquid metal (McDonough *et al.* [6]) or condensing vapor (Berenson [7]) which additionally served as a heating medium.

The first general stability criterion for boiling experiments was formulated by Adiutori [8]. As the result of rather qualitative reasoning it states that small deviations from an equilibrium point in the transition boiling regime can be compensated if the characteristic curve of heat supply to the system is steeper than the one of heat withdrawal through boiling. Independently, Stephan [9] (and later Kovalev [10]) derived the same criterion in a more rigorous fashion and formulated it as an explicit function of the parameters of the experimental set-up. The first attempt to stabilize a purely electrically heated boiling process was Peterson's and Zaalouk's [11] temperature control of a platinum wire heated by electric current. The temperature was measured via the temperature

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NOMENCLATURE

a	thermal diffusivity [$m^2 s^{-1}$]
c_P	specific heat [$kJ kg^{-1} K^{-1}$]
f_C	corner frequency [Hz]
G	transfer function
K_C	controller gain [$W m^{-2} K^{-1}$]
K_1	slope of boiling curve [$W m^{-2} K^{-1}$]
K_2	y-intercept of linearized boiling curve [$W m^{-2}$]
l	length of heater [m]
q	heat flux [$W m^{-2}$]
q_V	volumetric heat source [$W m^{-3}$]
q_W	heat flux corresponding to setpoint w [$W m^{-2}$]
r	radius of the heater [m]
s	frequency variable [$rad s^{-1}$]
t	time [s]
T	temperature [K]
T_∞	temperature of stabilizing fluid [K]
T_S	saturation temperature of boiling fluid [K]
U	voltage [V]
w	saturation superheat setpoint [K]
z	spatial coordinate [m].

Greek symbols

α_1	heat transfer coefficient perimeter of heater [$W m^{-2} K^{-1}$]
α_2	heat transfer coefficient back of heater [$W m^{-2} K^{-1}$]
γ	(abbreviation)
η	imaginary part
ζ	real part
λ	thermal conductivity [$W m^{-1} K^{-1}$]
ρ	density [$kg m^{-3}$]
θ	saturation superheat temperature ($= T - T_S$)
θ_∞	saturation superheat of stabilizing fluid ($= T_\infty - T_S$)
τ_I	controller rate constant [s]
τ_{TC}	thermocouple time constant [s].

Subscripts

0	initial condition
B	boiling
C	controller
F	filter
i	thermocouple i
i	inner
M	measurement
MIN	minimum
MAX	maximum
o	outer
TC	thermocouple.

Superscripts

$\bar{\ }$	Laplace transformed variable.

dependence of the wire's electrical resistance and compared to the setpoint temperature, then the voltage was adjusted proportionally. The implemented controller allowed stable steady-state operation in all boiling regimes; however, no information was provided about stable regions of the temperature feedback gain. Johannsen and Kleen [12] provided the first detailed stability analysis of an actively controlled system with indirect electric heating. They confirmed the stability of their experimental set-up by modeling heater, controller, and anti-noise filters with transfer functions and determining the roots of the closed loop system. No details are shared about the model of the thick cylindrical heater. Controller tuning took care to prevent distortion of boiling-induced temperature fluctuations by control action. Dhuga and Winterton [13] further examined the stability of an indirectly heated system under proportional temperature control. They modeled the heater as a second-order system and supplemented Adiutori's stability condition with an additional criterion accounting for the thermal inertia of the heater. It depends on the ratio of thermal conductivity to thickness of the heater and poses a lower bound to the slope of the transition boiling curve (note that the slope is negative) around which steady-state experiments can be stabilized. Kitamura *et al.* [14] were first to analyse a spatially distributed heater model with respect to controllability. They were first as well to explicitly incorporate the maximal power of the heating, in this case a radiating heat source, into their stability analysis. By an eigenvalue analysis of the distributed transfer function they examined the influence of heater thickness on stability under proportional feedback control. Auracher and Marquardt [15] and Auracher *et al.* [16] derived an upper stability bound for temperature controlled systems with proportional feedback control. They used the describing function method to determine the onset of limit cycle oscillations occurring for high feedback gains due to saturation of the power supply of the electric heating. With Bode diagrams of the open and closed loop system they identified values of the feedback gain which allowed the measurement of characteristic temperature fluctuations on the boiling surface free of interference by the control action.

Almost all issues which need to be considered when designing stable boiling experiments have been recognized at some point. However, a systematic approach to stability analysis has not been formulated. The main requirements of a flexible procedure are:

(1) Assess the influence of all system components on stability, which includes the heater as a spatially distributed system, the controller, sensors and filters.

(2) Incorporate arbitrary control algorithms. So far, control has been limited to the proportional or proportional-integral type. Other control laws not only promise greater stability, but more complex algorithms might be necessary to accomplish more difficult control tasks. One such case could be servo-control of instationary boiling experiments where the wall superheat is driven according to a specified reference trajectory. Reported results of previous such experiments indicate the inadequacy of PID-type controllers [17].

(3) Utilize design degrees of freedom to improve stability in a systematic fashion.

(4) Quantify the effects of nonideal system behavior such as controller saturation or the global nonlinearity of the boiling curve.

Accordingly, the main part of the paper is organized into three parts: the basic mathematical procedure of determining system stability is outlined and demonstrated for a simple example. By means of a case study it is then highlighted how the suggested stability analysis can be applied to the systematic design of an experimental apparatus. At last, limitations of the approach arising from nonideal system behavior in real experiments are quantified.

2. STABILITY ANALYSIS—A TUTORIAL EXAMPLE

The basis for the semi-analytic stability analysis outlined below are linear models of all components of the stabilization scheme. Therefore, the models and their fundamental assumptions are defined first. Then the actual stability evaluation is carried out resulting in the derivation of parameter dependent regimes of stable system behavior.

2.1. *Component modeling*

The elements making up the control loop for stabilization of a boiling system are schematically shown in Fig. 1 (top). In the case of passive stabilization (i.e. using a stabilizing fluid) only the heater is retained. The individual components are described by linear models in the frequency domain. As heat conduction is described by partial differential equations, the transfer function of the heater consequently is spatially distributed. The following steps follow the procedure for stability analysis of general distributed parameter systems described in [18].

2.1.1. *Heater*. For the purposes of this study we assume a one-dimensional (1D) temperature field in a cylindrical heater as depicted in Fig. 1 (bottom left). As transient three-dimensional (3D) finite element simulations of cylindrical heaters have shown, the assumption of a 1D temperature distribution is fair for common heater geometries and high conductivity materials like copper, even if energy is supplied to the cylinder along its curved surface. In its general form, the model must account for heat input from a heat source $q_{Hi}(t)$ and from a stabilizing fluid (heat transfer coefficient α_i) along the perimeter of the cylinder (index "1") and at $z = 0$ (index "2"), heat withdrawal by boiling $q_B(t)$ at $z = l$, as well as heat generation per unit volume $q_V(t)$ by ohmic heating. Note that $q_{Hi}(t)$ or α_i can also take on negative values to account for heat losses. For a one-dimensional system, heat transfer across the boundary and internal heat generation are mathematically identical, so that $q_{H1}(t)$ and $q_V(t)$ can be lumped into a single source term $q_V(t)$ without loss of generality; $q_{H2}(t)$ will simply be denoted $q_H(t)$. The differential equation with boundary and initial conditions reads as

$$\rho c_P \frac{\partial \theta}{\partial t} - \lambda \frac{\partial^2 \theta}{\partial z^2} = \frac{2\alpha_1}{r}(\theta_\infty - \theta) + q_V(t) \quad z \in (0, l) \tag{1}$$

$$-\lambda \left.\frac{\partial \theta}{\partial z}\right|_{z=0} = q_H(t) + \alpha_2(\theta_\infty - \theta(z=0,t)) \tag{2}$$

$$-\lambda \left.\frac{\partial \theta}{\partial z}\right|_{z=l} = K_2 + K_1 \theta(z = l, t) \tag{3}$$

$$\theta(z, t=0) = \theta_0(z) \quad z \in [0, l]. \tag{4}$$

The material properties of the heater are assumed to be independent of temperature. The boiling curve is approximated linearly around the operating point with K_1 as the slope of the boiling curve (cf. Fig. 1, bottom right). All temperatures θ represent the superheat above the boiling fluid's saturation temperature.

The transfer function model $\bar{\theta}(z, s)$ of the heater is obtained by Laplace transform of (1)–(4) using the Green's function method [19]:

$$\begin{aligned}\bar{\theta}(z,s) = {} & (\gamma^2(-(K_1\alpha_2 + \gamma^2\lambda^2) \\ & \times \sinh \gamma l - \gamma\lambda(\alpha_2 + K_1)\cosh \gamma l))^{-1} \\ & \times \Bigg[\Bigg((-K_1 \sinh \gamma(l-z) - \lambda\gamma \\ & \times \cosh \gamma(l-z))(\alpha_2 \cosh \gamma z \\ & + \gamma\lambda \sinh \gamma z - \alpha_2)\left(\frac{2\alpha_1\theta_\infty}{r\lambda s} + \frac{\bar{q}_V}{\lambda}\right)\Bigg) \\ & + \Bigg((\alpha_2 \sinh \gamma z + \lambda\gamma \cosh \gamma z) \\ & \times (K_1 - K_1 \cosh \gamma(l-z) - \gamma\lambda \\ & \times \sinh \gamma(l-z))\left(\frac{2\alpha_1\theta_\infty}{r\lambda s} + \frac{\bar{q}_V}{\lambda}\right)\Bigg) \\ & + \gamma^2\left(-\left(\bar{q}_H + \frac{\alpha_2\theta_\infty}{s}\right)(K_1 \sinh \gamma(l-z)\right.\end{aligned}$$

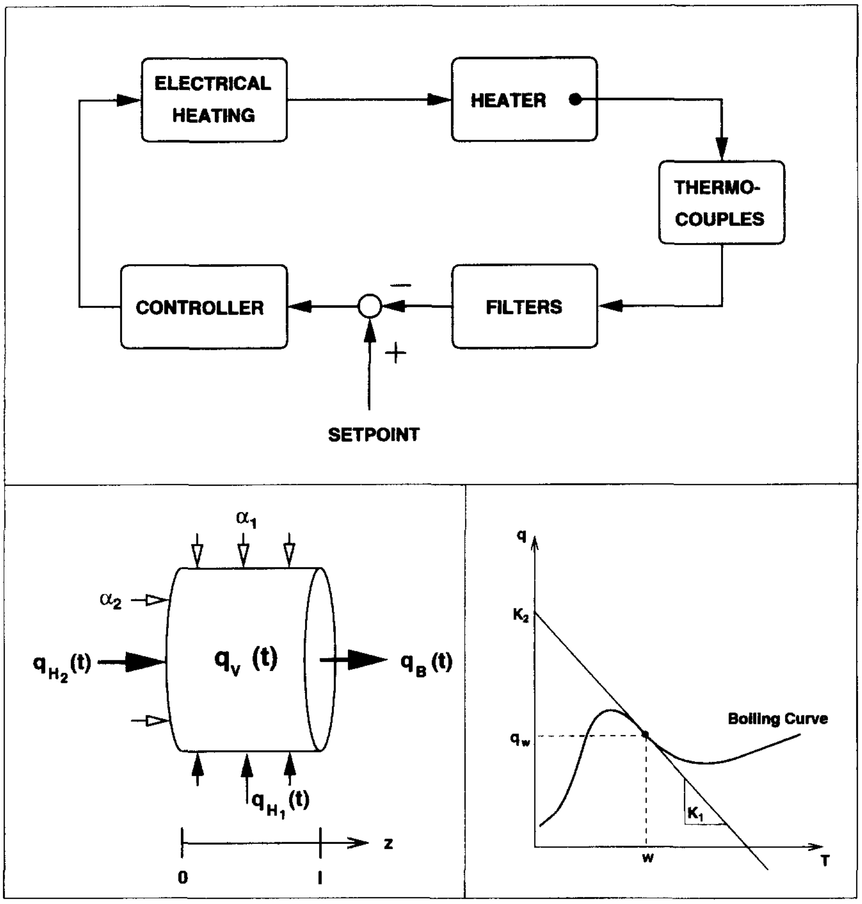

Fig. 1. Control loop elements (top). Cylindrical heater with boundary conditions (bottom).

$$+\gamma\lambda\cosh\gamma(l-z))$$

$$+\frac{K_2}{s}(\alpha_2\sinh\gamma z+\gamma\lambda\cosh\gamma z)\bigg)\bigg] \quad (5)$$

where

$$\gamma=\gamma(s)=\sqrt{\left(\frac{\rho c_P sr+2\alpha_1}{r\lambda}\right)}. \quad (6)$$

It describes the temperature distribution in a heater of homogeneous material properties. Similar expressions can be derived in a straightforward manner for composite heaters made of several layers of different material and geometric properties or different heat transfer and generation characteristics.

2.1.2. *Controller.* The controller determines the input to the heater from the measured, and possibly filtered, heater temperatures and the setpoint value w for the wall temperature superheat:

$$\bar{q}_H(s)=\int_0^l G_C(z',s)[\bar{w}(z',s)-\bar{\theta}_F(z',s)]\,dz'. \quad (7)$$

Normally, not a temperature profile, but only temperatures at discrete locations $z_{M,i}$ of the heater can be recorded. The general linear controller transfer function can therefore be written:

$$G_C(z,s)=\sum_i G_{C,i}(s)\delta(z-z_{M,i}) \quad (8)$$

2.1.3. *Thermocouples and filters.* The dynamics of thermocouples can be approximated by a first-order lag transfer function:

$$G_{TC,i}(s)=\frac{\bar{\theta}_{M,i}(s)}{\bar{\theta}(z_{M,i},s)}=\frac{1}{1+\tau_{TC,i}s}. \quad (9)$$

Filters are used to damp the effect of high frequency noise and low frequency boiling-induced temperature fluctuations on control action. Their dynamics can be described by linear transfer functions $G_F(s)$.

The transfer function of the closed loop system can be obtained by combining the transfer functions of the individual control loop components.

2.2. *Stability analysis*

For the tutorial example as well as for the case study in the next section, we will study the stability of a cylindrical copper heater of length $l = 10$ mm and radius $r = 17.5$ mm. For now, the copper block is assumed to be indirectly heated at $z = 0$ with no additional stabilizing fluid or direct (ohmic) heating. The temperature at $z = l$ is measured instantaneously

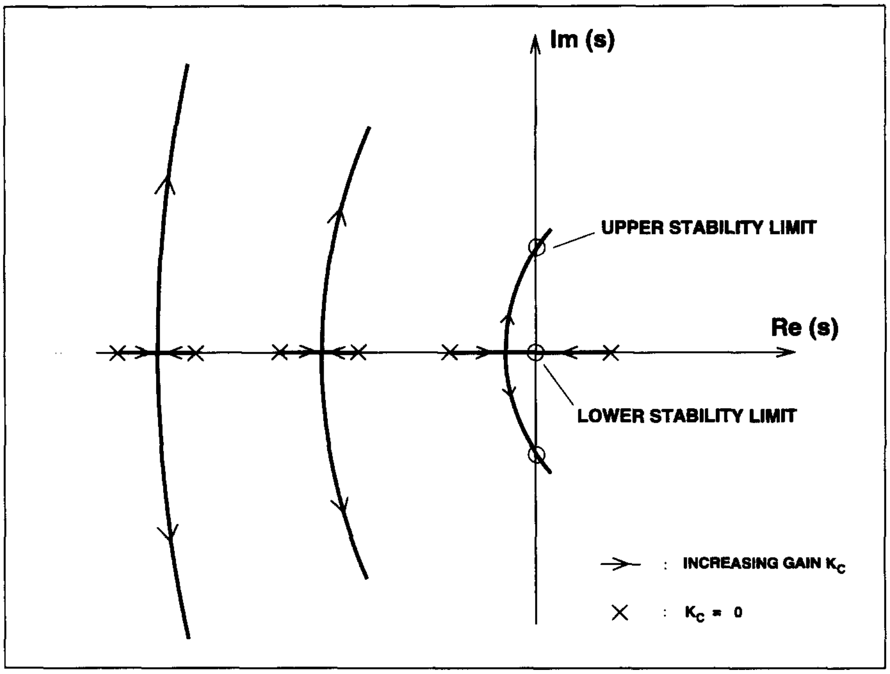

Fig. 2. Root locus of a proportionally controlled heater in transition boiling.

(thermocouple time constant $\tau_{TC} = 0$ s) and controlled solely by proportional feedback (gain K_C). There are no filter elements in the control loop. In this case, combining the transfer functions of the heater, equation (5), and the controller, equations (7) and (8), the characteristic equation

$$\sqrt{\frac{s}{a}}\lambda \sinh\left(\sqrt{\frac{s}{a}}l\right)+K_1 \cosh\left(\sqrt{\frac{s}{a}}l\right)+K_C = 0 \qquad (10)$$

can be determined in analogy to lumped parameter systems [18].

A system is asymptotically stable if all roots of the characteristic equation possess a negative real part. Equation (10) is transcendent and has got an infinite number of complex roots. Their location, the root locus, is shown in Fig. 2 for a system with negative K_1, i.e. in the transition boiling regime, in dependence on the proportional gain $K_C > 0$. The branch of the root locus dominating stability is the one crossing the origin and the imaginary axis. The root locus shows that for $K_C = 0$, the uncontrolled system, one root has a positive real part rendering the whole system unstable. With increasing K_C the root locus crosses the imaginary axis twice: at the origin, the lower stability bound, and again at a pair of purely imaginary roots, the upper stability bound. The existence of two stability bounds is a characteristic for actively controlled heaters in the transition boiling region. If K_1 were positive, the root $K_C = 0$ would be located in the left half plane, thus no lower stability bound existed.

2.2.1. *Lower stability bound.* Since at the lower stability bound the root locus crosses the origin, it can easily be obtained by setting $s = 0$ in the characteristic equation (10):

$$K_C = -K_1. \qquad (11)$$

This result complies with the stability criterion of Adiutori [8] and Stephan [9]. Their stability bound was determined from the condition

$$\frac{\mathrm{d}q_H}{\mathrm{d}\theta} = \frac{\mathrm{d}q_B}{\mathrm{d}\theta}. \qquad (12)$$

Using $q_H = K_C(w-\theta)$ and $q_B = K_2+K_1\theta$, this results in (11).

2.2.2. *Upper stability bound.* The parameterization

$$\sqrt{\frac{s}{a}}l = \zeta+\mathrm{i}\eta; \quad \zeta,\eta\in\mathbb{R} \qquad (13)$$

is chosen for the upper stability bound according to Fig. 2. Inserting this expression into the characteristic equation (10), taking advantage of the fact that the real part vanishes at the stability limit, and splitting (10) into its real and imaginary part, the following system of linear equations is obtained:

$$\begin{bmatrix}\cosh\zeta\cos\zeta & 1\\ \sinh\zeta\sin\zeta & 0\end{bmatrix}\begin{bmatrix}K_1\\ K_C\end{bmatrix} = \begin{bmatrix}\frac{\lambda}{l}\zeta(\cosh\zeta\sin\zeta-\sinh\zeta\cos\zeta)\\ -\frac{\lambda}{l}\zeta(\sinh\zeta\cos\zeta+\cosh\zeta\sin\zeta)\end{bmatrix}. \qquad (14)$$

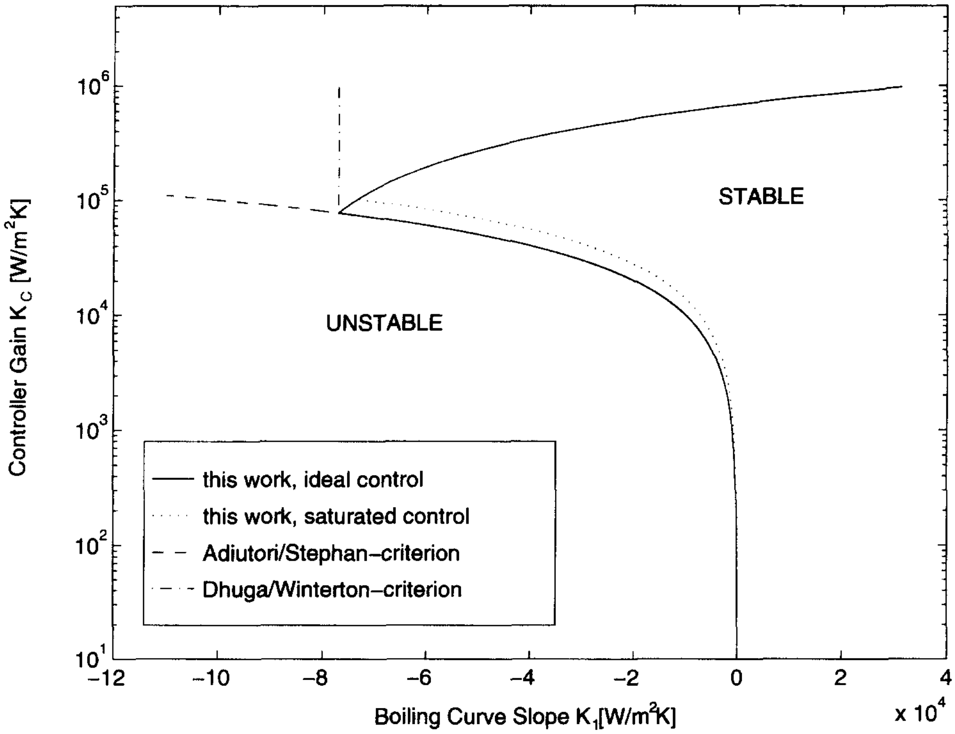

Fig. 3. Stability diagram for a proportionally controlled heater.

By solving (14) for different values of ζ, parameter pairs (K_1, K_C) forming the stability limit can be computed.

Figure 3 shows the obtained stability limits in the (K_1, K_C)-plane. The stability diagram allows to predict which operating points, associated with a boiling curve slope K_1, can be stabilized for a given controller gain K_C in steady-state experiments. For $K_1 > 0$ (nucleate and film boiling) the stable region is bounded only by the upper limit. For $K_1 < 0$ (transition boiling) an upper and lower limit exists. The greater the slope of the boiling curve in the transition boiling regime, the narrower the range of admissible controller gains. Finally, for slopes lower than a certain $K_{1,MIN}$, stabilization is not possible at all. This is consistent with the finding of Dhuga and Winterton [13] who derived

$$K_{1,MIN} = -\frac{2\lambda}{l} \tag{15}$$

from a simplified model of a proportionally controlled heater. It can also be concluded from Fig. 3 that stabilization of all boiling regimes is possible with just one controller gain ($\approx 10^5$). Note that the stability of the apparatus only depends on λ, l, K_1 and K_C [equation (14)]. ρ, c_P or the absolute level of boiling heat transfer K_2 do not have any effect on stability.

The results of this analysis were confirmed by application of the Nyquist stability criterion and dynamic simulation of the controlled heater.

3. SYSTEMATIC STABILIZATION OF AN EXPERIMENTAL APPARATUS

The outlined procedure allows the systematic design of an experimental apparatus with respect to stability. In the following, some important results obtained using the detailed stability analysis are highlighted. They include a fundamental comparison between active and passive stabilization schemes, choice of optimal controller type and settings, and the effect of additional control loop elements like thermocouples and filters on stability. A more detailed analysis including geometric and material considerations, more complex control configurations as well as an experimental verification of the results will be the subject of a forthcoming study.

We will look at a cylindrical copper heater of radius $r = 17.5$ mm and length $l = 10$ mm. Its thickness is chosen rather large to ensure uniform boiling conditions across the boiling surface.

3.1. *Passive or active stabilization*

Four types of stabilization are studied: stabilization by a fluid at $z = 0$ (a) or along the curved surface of the cylinder (b), and alternatively an active stabilization scheme by temperature control of the boiling wall temperature. In the latter case, either the energy input at $z = 0$ [indirect electrical heating, (c)] or by internal heat generation [direct electrical heating, (d)] is adjusted. To study the basic differences in the alternative methods of stabilization, it is assumed that for electrical heaters the controller is purely proportional (gain constant K_C), that the thermocouples measure the wall temperature at $z_M = l$ with no lag, and that there are no filter elements in the control loop. Figure 4 shows the obtained stability limits in the K_1/α- [(a) and (b)] and the K_1/K_C [(c) and (d)] parameter plane. Clearly, the most favorable method of stabilization is temperature control by direct electrical heating (d): arbitrary negative slopes K_1 in the transition boiling regime can be chosen as stable steady-state operating points; in contrast to temperature control via indirect heating (c) the 'upper' stability limit has a negative slope. The stability curve for indirect heating is the same as the one studied in Fig. 3, only this time K_C is not plotted logarithmically. If stabilizing fluids are used (a, b), heating along the curved surface of the cylinder is preferable. Then, arbitrarily steep boiling curves can be recorded. If the stabilizing fluid is interfacing the heater at its back-side opposite the boiling surface ($z = 0$), a lower bound for operable K_1s exists. The corresponding stability condition for the use of stabilizing fluids is

$$K_1 = -\lambda \left[\frac{\alpha_2 + \sqrt{\left(\frac{2\alpha_1\lambda}{r}\right)}\tanh\left(\sqrt{\left(\frac{2\alpha_1}{r\lambda}\right)}l\right)}{\lambda + \alpha_2\sqrt{\left(\frac{r\lambda}{2\alpha_1}\right)}\tanh\left(\sqrt{\left(\frac{2\alpha_1}{r\lambda}\right)}l\right)} \right] \tag{16}$$

For the limiting cases $\alpha_1 = 0$ (a) and $\alpha_2 = 0$ (b) shown in Fig. 4, this equation reduces to

$$K_1(\alpha_2 = 0) = -\sqrt{\left(\frac{2\alpha_1\lambda}{r}\right)}\tanh\left(\sqrt{\left(\frac{2\alpha_1}{r\lambda}\right)}l\right) \tag{17}$$

$$K_1(\alpha_1 = 0) = -\frac{\lambda\alpha_2}{\lambda + \alpha_2 l}. \tag{18}$$

Relation (18) confirms the stability limit derived by Stephan [20] for this special type of fluid heating. For large values of α_2, equation (18) reveals the principal lower bound of K_1 for this, very common, experimental set-up:

$$K_{1,\mathrm{MIN}} = -\frac{\lambda}{l}. \tag{19}$$

For further stability analysis in the framework of this

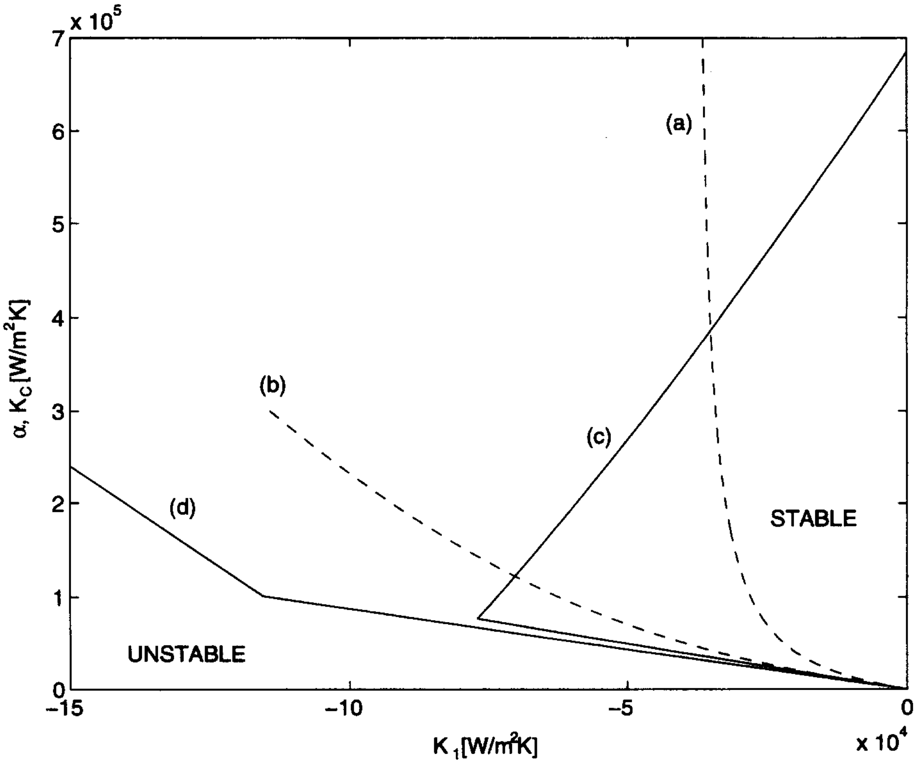

Fig. 4. Stability bounds for passive and active stabilization: stabilizing fluid at $z = 0$ ($\alpha_1 = 0$, $\alpha_2 \neq 0$), (b) along perimeter ($\alpha_1 \neq 0$, $\alpha_2 = 0$); temperature control by (c) indirect electrical heating, (d) direct electrical heating.

case study and as a scheme adequate for the type of heater studied, active temperature control is chosen as the method of stabilization, mainly because of its flexibility:

(1) A systematic recording of the boiling curve can easily be achieved simply by repeatedly adjusting the setpoint temperature of the controller. With fluid stabilization the operating point has to be adjusted by increasing the temperature of the heating medium or by a separate means of direct or indirect heating. The first is not practical if transient experiments are to be conducted because of the slow dynamics of the heating process. In the latter case, if an adjustable heat source needs to be implemented in any case, the (cheap) installation of a temperature sensor and a controller is preferable to the (costly) apparatus necessary for the circulation of a stabilizing fluid.

(2) Taken aside the principal bound on operable K_1s [equation (19)], the experimental effort to achieve heat transfer coefficients necessary for passive stabilization is high. The recording of water boiling curves, for example, often requires the use of liquid metals as a stabilizing fluid [6]. Fluid heating all across the surface of the heater [Fig. 4(b)] is prohibitive in most cases; if passive stabilization is implemented, method (a) is employed, mostly using thin tubes heated from the inside by a flowing liquid or condensing vapor [20]. In this case, for a tube of inner radius r_i and outer radius r_o, equation (18) can be employed using

$$l = r_o - r_i \quad \left(\text{if } \frac{r_o - r_i}{r_i} \ll 1\right).$$

Assuming that heat transfer coefficients maximally achievable are in the range around 40 kW m^{-2} K [20], an arrangement of low thermal resistance, e.g. copper tubes ($\lambda \approx 385$ W m^{-1} K^{-1}) with wall thickness $l = 0.5$ mm, would yield a stabilizable transition boiling slope value $K_1 \approx 38$ kW m^{-2} K according to equation (18), a small value compared to the potential of electrical heating shown in Fig. 4. This means that for practical purposes passive stabilization is inferior to active schemes not only in terms of required experimental effort, but also with regard to achievable stability.

(3) If a variety of boiling fluids is to be examined without costly changes in the experimental design, active temperature control clearly is the most flexible approach.

Although Fig. 4 shows that direct heating yields a larger, unbounded region of stability in transition boiling, here, indirect heating is chosen for its ease of implementation. It should be noted that ohmic direct heating and circumferential heating (e.g. by a heating wire coiled around the cylinder [16]) display the same stability characteristics as they both yield mathematically identical source terms in equation (1). This is, of course, a consequence of the particular modeling approach chosen here, but reflects the behavior of real heaters closely, too.

3.2. *Choice of controller*

So far, only proportionally controlled heaters have been studied. One disadvantage of purely proportional controllers is the characteristic steady-state offset. For removal of the offset, a proportional-integral (PI) control law should be applied:

$$G_C(s) = K_C\left(1 + \frac{1}{\tau_1 s}\right). \tag{20}$$

Figure 5 compares the stability characteristics of PI- and P-controllers. It is evident that integral control action means a loss of stability. For suitable values of $\tau_1 \approx 0.5$ s the decrease of $K_{1,\mathrm{MIN}}$ is about 40 kW m^{-2} K^{-1}, cutting the transition boiling operating region by one-half. If the steady-state offset of proportional control is of no concern, for the mere recording of a stationary boiling curve it is not, it should thus be preferred to PI-control. PID-controllers are not advisable since derivative control action is sensitive to measurement noise; if filters are used for noise reduction, an additional loss of stability is caused as will be demonstrated below.

It should be stressed that not the whole feasible (i.e. stable) range of controller gains for a given boiling curve slope K_1 can actually be implemented, if boiling-induced fluctuations of heat flux or temperature are to be resolved by the measurements. In this case the stabilization should not distort the boiling characteristics by control action. To assess the distortion of boiling by temperature control, a comparison of the frequency response of the uncontrolled and the controlled system was suggested in [15]. In general, high control gains tend to cause large distortions in the frequency range of interest.

3.3. *Effect of thermocouple and filter dynamics*

The effect of additional control loop elements like thermocouples and filters on stability is often neglected. Since a thermocouple can be thought of as a simple filter, we will compare three stability diagrams for the proportionally controlled heater: for an 'ideal' thermocouple with time constant $\tau_{TC} = 0$ s, for a lag constant $\tau_{TC} = 0.05$ s, and for a thermocouple additionally serving as a filtering element. For the latter, the time constant is determined so as to filter out temperature fluctuations above a corner frequency of $f_C = 1$ Hz:

$$\tau_{TC} = \frac{1}{2\pi f_C} = 0.16 \text{ s}. \tag{21}$$

The temperature is measured at $z_M = l$, and pure proportional control is employed. Figure 6 shows the results. The filtering thermocouple causes a loss of stability $\Delta K_{1,\mathrm{MIN}}$ of about 30 kW m^{-2} K^{-1}. Even the regular thermocouple reduces the margin by a significant 10 kW m^{-2} K^{-1}. As a consequence, the lag introduced by thermocouples and filters should be kept at the necessary minimum for optimal stability.

The examples above demonstrate that advantages

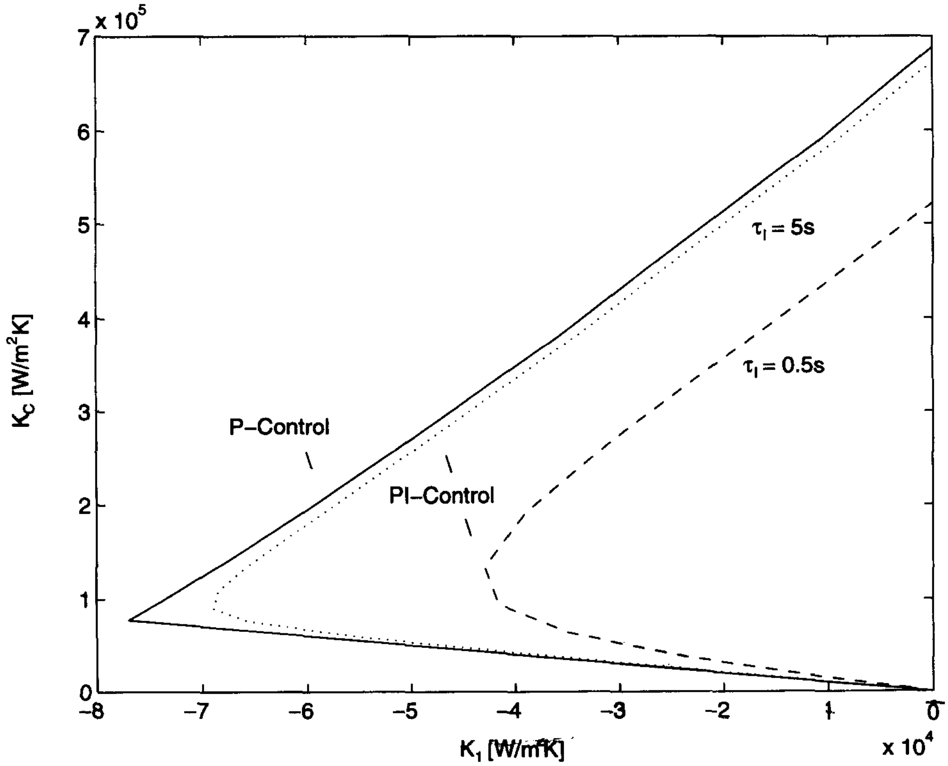

Fig. 5. Comparison of P- and PI-control stability regimes.

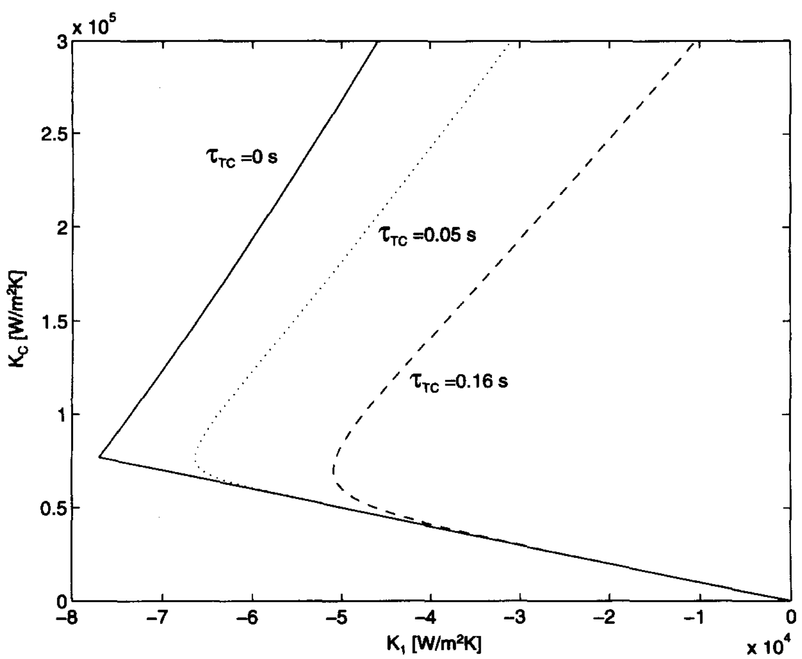

Fig. 6. Loss of stability by time-lag elements.

and disadvantages of alternative stabilization schemes can be quantified and studied systematically using the procedure outlined in Section 2.

4. RESTRICTIONS ARISING FROM REAL EXPERIMENTS

In real experiments the following nonideal effects have to be considered:

(1) The controller output, the heat supply to the system for active stabilization, is subject to constraints. Often, a voltage-controlled power supply is used, rendering the lower limit $q_{H,MIN} = 0$ W m^{-2} (power off) and $q_{H,MAX} = f(U_{MAX})$ (maximum voltage).

(2) The boiling curve is not linear, as assumed for the previous stability analysis, but has got the well known highly nonlinear shape.

As a result, the linear, 'ideal', stability analysis of the previous sections has to be modified to account for the effect of the nonlinearities. In the following, the reduction of the operable range of boiling curve slopes due to the limitation of the power supply is quantified by means of simple steady-state considerations. Then the dynamics of the nonlinear system, in particular temperature oscillations occurring for high feedback control gains, are examined by a numerical stability analysis.

As a test system, the experimental set-up already examined in Section 3 is used, i.e. an electrically heated cylindrical copper block of radius $r = 17.5$ mm and length $l = 10$ mm. The wall temperature is measured at $z_M = l$ and is controlled purely proportionally.

4.1. *Limitations of the power supply*

The presence of saturation limits to the controller output brings about two effects. The first one is that not the whole region of stability found by the outlined procedure can be implemented since parts of the stability region require control action outside of the bounds of the power supply. To assess this limitation quantitatively, an expression for the stationary heat flux through the system has to be derived. Around the operating set point of the boiling curve (w, q_W), cf. Fig. 1, for proportional control, it can be determined by equating the stationary controller output (heat flux *into* the heater) with the stationary boiling heat transfer (heat flux *from* the heater):

$$q_{STAT} = \underbrace{K_l(\theta_{STAT}(z=l) - w) + q_W}_{\text{heat flux to the fluid}} = \underbrace{K_C(w - \theta_{STAT}(z=l))}_{\text{controller output}}. \quad (22)$$

θ_{STAT} is the steady-state saturation superheat temperature of the heater's boiling surface. From equation (22) the stationary heat flux can be determined as:

$$q_{STAT} = \frac{K_C}{K_l + K_C} q_W. \quad (23)$$

We also obtain the steady-state offset of the proportional controller:

$$w - \theta_{STAT}(z=l) = \frac{1}{K_l + K_C} q_W. \quad (24)$$

Note that equation (23) yields a physical interpretation of the lower stability bound: at $K_C = -K_l$ the necessary heat supply becomes infinite. For controller gains close to the stability limit $-K_l$ both the required heat supply and the control offset grow very large. In order to keep the steady state controller output within its bounds, the following conditions have to be satisfied:

$$0 \leqslant q_{STAT} \leqslant q_{H,MAX}. \quad (25)$$

By substituting equation (23) into (25) the following two requirements on the feedback gain K_C are derived:

$$K_C \geqslant -K_l \quad (26)$$

$$K_C \geqslant -K_l \frac{q_{H,MAX}}{q_{H,MAX} - q_W}. \quad (27)$$

For negative slopes K_l, equation (27) is the limiting requirement for K_C. It shows that Adiutori's and Stephan's stability criterion (as formulated for temperature-controlled experiments by Dhuga and Winterton [13]) is not correct for real experiments but only a limiting case for an infinite power supply, in which case equations (26) and (27) become identical. In Fig. 3 the true lower bound for the parameters $q_W = 140$ kW m^{-2}/$q_{H,MAX} = 576$ kW m^{-2} K^{-1} (see below) is indicated by a dotted line.

4.2. *Limit cycle oscillations*

Beside the limitation of the theoretically possible region of stability, the presence of saturation limits also causes oscillations of the controlled wall temperature for high control gains. The phenomenon of limit cycle oscillations is well known from nonlinear dynamics and systems theory [21]. It has been shown that stable limit cycle oscillations can arise if a complex conjugated pair of eigenvalues of the linearized system crosses the imaginary axis at a so-called Hopf bifurcation point, as is the case at the upper stability bound in Fig. 2. For the operation of boiling experiments this would mean that with increasing controller gain a stable steady-state operating point would give way to nonlinear oscillations of the controlled temperature. This phenomenon has been observed for actively temperature-controlled experiments before [22]; it has not been determined, however, whether these oscillations are stable and of which magnitude their amplitude and frequency are. For the stability analysis of this type of nonlinear system only approximate analytical tools are available [16]. We used the software package AUTO [23] that allows an exact numerical bifurcation and stability analysis of nonlinear systems.

Apart from the saturation limit of the controller output, the results of the analysis depend on the operating point (w, q_W) on the nonlinear boiling curve. It was chosen as the steepest point on the transition boiling curve of the fluorinert C_6F_{14}(FC-72, 3M-Company) at atmospheric pressure measured in pool boiling experiments on a horizontal thick copper heater ($q_W = 140$ kW m^{-2}, $w = 34.8$ K, $K_l = -7.3$ kW m^{-2} K^{-1}, $q_{H,MAX} = 576$ kW m^{-2} K^{-1}).

Figure 7 shows amplitude and frequency of the stable limit cycle oscillations occurring as the controller gain is increased beyond the upper stability limit from a starting point in the stable stationary operating region. Near the upper stability bound the frequency is at its maximum just below 2 Hz. The amplitude of the temperature oscillations never exceeds 0.5 K.

An important practical result of the performed nonlinear stability analysis is that although limit cycle oscillations generally should be avoided, they are stable with moderate amplitudes and thus allow safe conduction of the boiling experiment.

5. LIMITATIONS OF UNIFORM BOILING ASSUMPTIONS

The assumption underlying the outlined stability analysis is a uniform temperature distribution across the boiling surface. As indicated in the introductory section, there are circumstances which knock over this assumption. For one, thin and long heaters tend to exhibit nonuniform boiling states across their surface. In experiments using electrically heated wires which employed temperature control by measuring the (temperature-dependent) electrical resistance of the wire, stable coexistence of nucleate and film boiling along the wire was observed [12, 22]. If the apparatus is not actively controlled, this also applies to tubes heated by the flow of a stabilizing fluid, suitable disturbances cause the displacement of nucleate by film boiling in a wave-like fashion or vice versa, depending on the level of heat supply [24]. A second experimentally observed phenomenon discrediting the uniform boiling assumption is the formation of dry patches in the nucleate boiling region prior to the critical heat flux [2, 25] for both thick and thin heaters. If the expansion of some of these vapor patches were the dominating mechanism leading to the post-CHF instability of boiling, as argued by Ouwerkerk [25] or lately by Unal *et al.* [3], then the stability of nonuniform temperature distributions would play an important role for all types of boiling experiments.

Theoretical studies of the stability of nonuniformly boiling systems have been conducted by various researchers. Most examined the stability of coexisting boiling regimes on electrically heated wires; they derived conditions on the disturbances necessary for transition to nonuniform behavior [26], worked out the dependence on the type of electrical power supply [24], and derived expressions for the velocity and width of the resulting temperature wave fronts [27].

In our opinion, the following issues relating to the

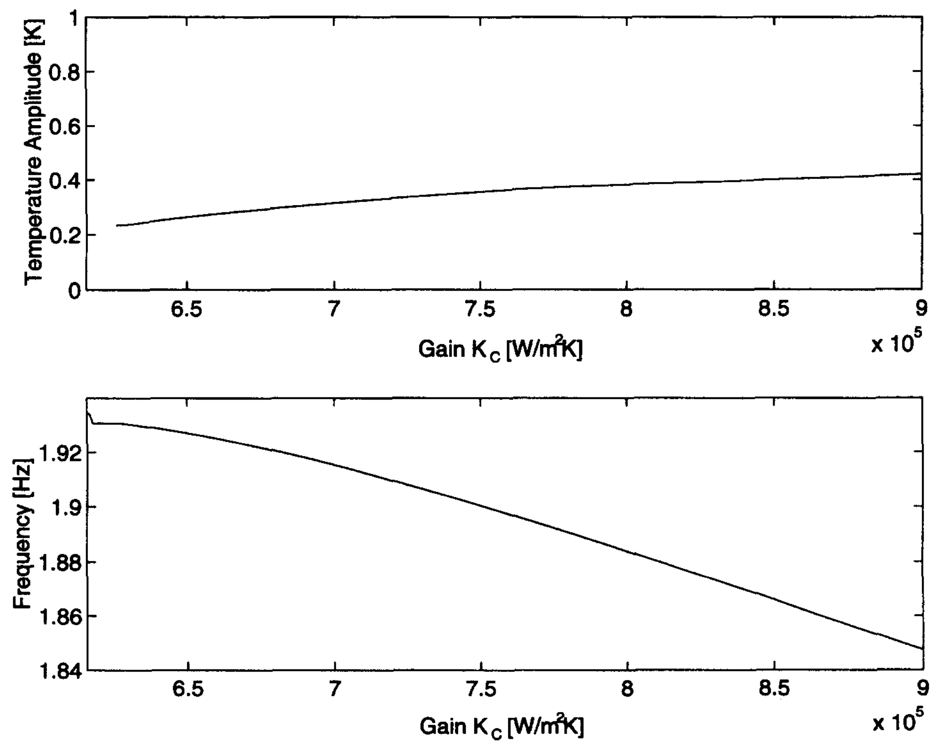

Fig. 7. Amplitude and frequency of limit cycle oscillations in dependence on controller gain K_C.

stability of nonuniformly boiling systems still need to be addressed:

(1) It is not clear whether stable nonuniform boiling can exist in arbitrary experimental arrangements (e.g. thick heaters of the type studied in this paper). Recently, Haramura [28] derived conditions for stability of both thick and thin heaters in the presence of nonuniform temperature distributions on the boiling surface. Under certain conditions, he concluded stable transition boiling for heat flux controlled, i.e. un-stabilized, experiments. This contradicts all stability findings derived so far and in this paper. The question remains how nonuniform boiling is possibly initiated, whether it can be artificially stabilized by an unfortunate control scheme, and how it can be avoided by systematic design changes.

(2) The true mechanism for burnout in heat flux controlled systems is not yet fully understood. If it can be explained by the formation of dry patches prior to CHF, a thorough understanding of their stability, expansion and disappearance is crucial. Then, hydrodynamic effects in the liquid layer adjacent to the boiling surface will have to be taken into consideration, too.

6. SUMMARY

A rigorous approach for the stability analysis of boiling systems under the assumption of uniform boiling has been suggested. Following this procedure, the stability characteristics of actively (temperature control by electrical heating) or passively (use of a stabilizing fluid) stabilized systems can systematically be compared. All stability criterions from previous studies have been confirmed or generalized. The upper stability bound for actively controlled systems, previously observed in experiments, can be determined exactly.

Aside from *a posteriori* analysis of existing systems, the suggested procedure allows the systematic design of an apparatus maximizing the regime of stability. For active stabilization, pure proportional control assures optimal stability. Filtering elements reduce the margin of stability unnecessarily. The controller output constraints present in real experiments lead to a shift in the implementable lower stability limit. They also lead to limit cycle oscillations of the wall temperature for high controller gains. Amplitude and frequency of the oscillations can be determined using numerical algorithms for stability and bifurcation analysis. It is shown that the resulting fluctuations are stable and well in the range of safe operation for typical experimental set-ups.

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